# FEMLIP: the twin method of MPM with a validated integration scheme. Several applications to non-Newtonian fluid flows and geomechanical problems

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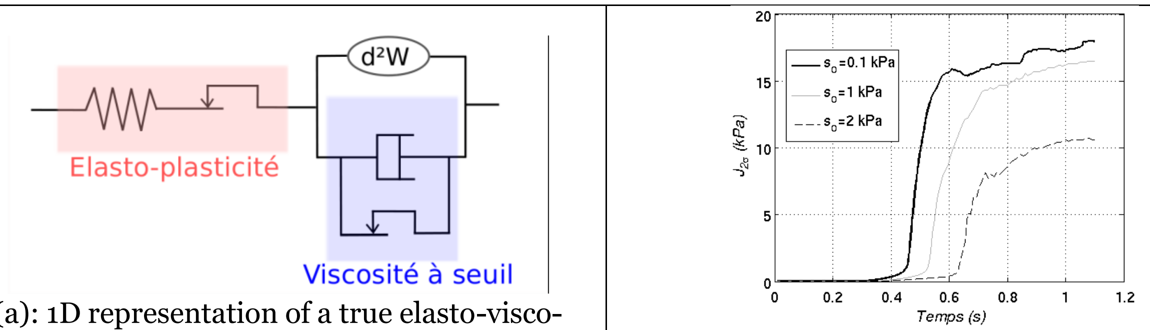
***Keywords:*** FEMLIP; Gaussian quadrature; complex fluid flow

On one hand, industrial processes may involve complex fluids often used for specific needs such as dip-coating, removal of objects embedded in those fluids, and, on the other hand, natural processes such as landslides, snow avalanches or mantle convection imply the coupling of mechanics with thermal or hydraulic phenomena. Therefore, in both cases, a better understanding of the behaviour of such fluids may help to better respond to the needs. In this context, a numerical method accounting for large deformation processes is required to follow in space and time the flow of an elasto-visco-plastic material, including thus history variables.

Based on the same pioneering work of Brackbil et al. (1988), two methods have been developed simultaneously: the Material Point Method (MPM) by Sulsky et al. (1994) and the Finite Element Method with Lagrangian Integration Points (FEMLIP) by Moresi et Solomatov (1995). Both methods use, on one hand, an Eulerian grid to compute the kinematic field and apply the boundary conditions and, on the other hand, a Lagrangian set of particles to carry history variables and in a given configuration to serve as integration points to compute the discretized set of equations to be solved. Based on the kinematic field, the particle positions are updated to provide a new configuration of the materials. The main difference lies in the particle weights used for the integration: they are constant in time and represent the physical mass of each particle in the MPM whereas they are recomputed in each configuration to check the Gaussian quadrature rules in the FEMLIP. Although being more physically sounded, this choice for the MPM is at the heart of many numerical instabilities which must be addressed in different ways as shown in the literature over the last 30 years. At the cost of being recomputed in each configuration (the numerical scheme is explicit for sparred mesh), the integration weights in the FEMLIP verify the Gaussian quadrature rules and polynomials are exactly integrated up to a given order (Moresi et al., 2003).

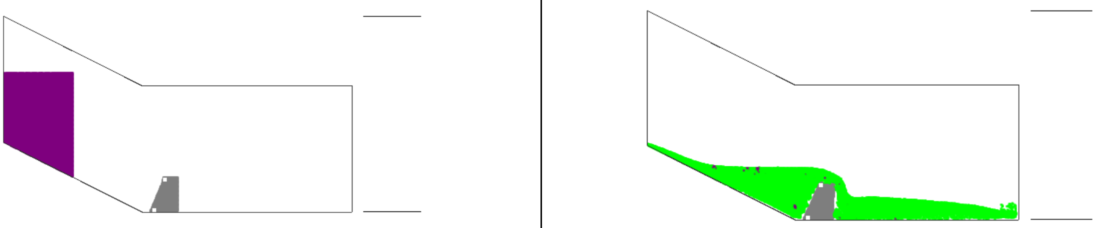
Over the last 25 years, the FEMLIP has been successfully applied to several problems in geophysics with visco-elastic materials (Moresi et al., 2002), in civil engineering with frictional boundary conditions (Dufour and Pijaudier-Cabot, 2005), in geomechanics with second order work criterion to skip from solid state to fluid one (Prime et al., 2014) and hydro-mechanical coupling (Li et al., 2016) and more recently in non-Newtonian industrial processes where the full capacity to handle elastic stresses in large deformation in contrary to the more classical regularized constitutive law in pure Eulerian FEM has proved to be highly more precise (Gueye et al., 2021).

For an illustrative purpose, in Figure 1 is presented an heuristic study of a landslide overtopping a retaining wall (Prime et al., 2014). In this model a true elasto-visco-plastic model was used with the second order work as the criterion to introduce the fluid behavior. Initially, before setting the gravity, the material behaves as a solid (purple color). Once the gravity is active, the material becomes a visco-elastic fluid (green color) and flows over the retaining wall. The second invariant of the stress tensor at the basis of the wall is tracked in time as an indicator of the force acting on the retaining wall for its design.



(a): 1D representation of a true elasto-visco-plastic model for the solid-fluid transition

(b): 2nd invariant of the stress tensor at the basis of the retaining wall



(c): Initial configuration of the landslide

(d): Final configuration

**Figure 1** Heuristic landslide analysis: (a) 1D representation of the 3D constitutive model, (b) force acting on the retaining wall, (c) the initial configuration with an unstable material and (d) the final equilibrium state when the flow has overtopped the retaining wall.

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## *Acknowledgements*

This work is a long lasting task which has involved many post-docs, PhD and master students. The author thanks them warmly for their fruitful and pleasant collaboration.